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A MATHEMATICAL MODEL OF THE PROCESSES OF FATIGUE WEAR AND DISINTEGRATION

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An analysis was conducted in [1, 2] into the behavior of the coefficients of stress intensity at the tips of subsurface cracks, located in an elastic overstressed half plane whose boundary is affected by normal and tangential contact stresses. These stress-intensity factors determine the development of cracks in an elastic medium and, thus, the fatigue quasibrittle destruction of bodies in contact with each other. Moreover, fatigue destruction of bodies depends on the level of material contamination and its resistance to crack formation.

In the present article we have laid out a statistical mathematical model for the processes of fatigue wear and disintegration, based on a study of a uniform mechanism for the development of fatigue cracks in quasibrittle materials.

1. The Suitability of Applying the Mechanics of Quasibrittle Destruction to the Study of Contact Fatigue. The main premise of the theory of fatigue destruction is the formation and the development of scattered microcracks, initiated by various defects (nonuniformities) in the material: microscopic pores, pitting, carbides, nonmetallic inclusions, etc. The process involved in the development of fatigue cracks around such defects is governed by the properties of the material and the stressed state of the material in the immediate vicinity of the defects, and this, in turn, depends on the normal and tangential stresses at the contact, as well as on residual stresses within the material.

The experimental and theoretical research [2-4] carried out to date enables us to isolate the fundamental factors characterizing fatigue destruction under loads which generate no significant plasticity phenomena in the material, and we have specific reference here to: normal and tangential contact stresses, residual stresses, the level of material contamination in the contact bodies and lubricants, the parameters of cyclical resistance to crack formation in the material, the structure of the material, etc.

Let us ascertain the possibility of utilizing the results obtained in the solution of contact problems for elastic bodies with cracks, based on the linear mechanics of quasibrittle destruction, insofar as this pertains to our study of the processes of contact fatigue.

1.1. Relative duration of crack generation and propagation phases. A variety of statements can be found in the literature, including those that are contradictory [5-7]. The assertion of the predominance of the generation phase, as a rule, is speculative in

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this case [5], while the opposite opinion is based on experimental data [6, 7] which demonstrate that the phase for the development of fatigue cracks consumes no less than 80-90% of the total time prior to destruction. In the following we will therefore make the assumption that the time to destruction coincides with the time required to propagate the fatigue cracks, and we will neglect the time of crack generation.

1.2. Initiators of destruction. Let us analyze this question by using steel as an example. In addition to the pitting and micropores, among the dangerous defects we should include the inclusions that are formed on a base of various chemical compounds. It has been established that the most frequent initiators of contact fatigue are the oxide inclusions and brittle carbides [8, 9]. This is explained by the lower level of temperature expansion for these inclusions in comparison to the steel matrix [3], which leads to the appearance about these inclusions of substantial tensile stresses and the rapid appearance of microcracks. Moreover, as shown by numerous experimental studies, the characteristic distances between the defects in the steel are large in comparison to their intrinsic values, thus making it possible, in studying the processes of fatigue, to use asymptotic methods and the results from [1, 2].

1.3. The microstructure of the material. According to the data from [10, 11], the boundaries of the ferrite grains, having dimensions of 1.2-48.5 μm, exerted no significant influence on the propagation of fatigue cracks.

Results have recently been obtained which indicate a positive effect from residual austenite on the fatigue lifespan of rocker bearings [8]. We can take this effect into consideration in the crack-resistance parameters included in the kinetic equation of crack development [4]. Information relative to the effect of alloying elements remains, for the moment, inadequate, and such as exists, is quite contradictory (see, for example, [8]) in order to be able to introduce this information into the fatigue-destruction model.

Thus, the microstructure of a material can be taken into consideration, at the present time, in the fatigue destruction model only in terms of its crack-resistance parameters.

1.4. The suitability of the mechanics of quasibrittle destruction, based on the dimension r_p of the plastic zone at the tip of the crack. According to [12], the radius r_p of the plastic zone at the tip of the crack in a plane stressed state is calculated in accordance with the formula

$$r_p = (1/6\pi)(k_1/\sigma_y)^2, \tag{1.4.1}$$

where k_1 is the coefficient of stress intensity for normal separation at the tip of the crack; σ_y is the yield stress of the material. Formula (1.4.1) is obviously valid only for the conditions under which the plastic zone at the tip of the crack is small in comparison with the half length ℓ of the crack, i.e.,

$$2r_p/l \ll 1. \tag{1.4.2}$$

The estimate (1.4.2) must be tested in both the development stage of relatively small cracks, on which most of the time prior to destruction is spent, as well as on the concluding stage of the destruction process, i.e., prior to the break, where $k_1 \approx K_{fc}$ (K_{fc} is the critical value of the coefficient of stress intensity at the tip of the crack under cyclical load, $K_{fc} < K_{Ic}$). For the first of these stages, with the aid of (1.4.1), we obtain

$$\frac{2r_p}{l} = \frac{1}{3\pi}\left(\frac{\tilde{q}}{\sigma_y}k_1'\right)^2 \leqslant \frac{1}{3\pi}(k_1')^2 \tag{1.4.3}$$

($\tilde{q}$ represents the maximum Hertz stress and k_1' is the dimensionless intensity factor: $k_1 = k_1'\tilde{q}\ell^{1/2}$). Inequality (1.4.3) follows out of the condition that there are no macroscopic plastic grains. We will now specify the coefficient of friction $\lambda = 0.01$ characteristic for the conditions of lubrication and we will also specify the angle of crack orientation $\alpha = \pi/2$ (i.e., the angle between the direction of the crack and the positive direction of the Ox axis in the system of coordinates coincident with the surface of the body). With such an α, as shown by the calculations in [1, 2], the maximum values of k_1' are attained under the condition that the residual stresses $q^0 = 0$. In view of the fact that the residual compression stresses exhibit a tendency to increase [13] as the process proceeds, the adopted assumption ($q^0 = 0$) corresponds, as a rule, to higher k_1'. The presence of compressive stresses in the surface layers of the material is associated with various aspects of the operation [13, 14]. Finally, bearing in mind the relatively weak dependence of k_1' on $\delta_0 = \ell/\tilde{b}$ ($\tilde{b}$ is the half-width of the Hertz contact region, or the minor semiaxis of the contact ellipse) and on the depth of crack penetration $y^{0'} = y^0/\tilde{b}$, we will specify the values of $\delta_0 = 0.1$ and $y^{0'} = -0.5$. The calculations from [1, 2] then yield $\max k_1' = 5.5\cdot10^{-5}$ and with the aid of (1.4.3) lead to the inequality $2r_p/\ell = 3.03\cdot10^{-9} \ll 1$. Estimate (1.4.2) is satisfied at this stage of the process.

When $k_1 = K_{Ic}$, for example, for the case of martensite aged steel ($K_{Ic} = 76$ MPa$\cdot$m$^{1/2}$, $\sigma_y = 1.8$ GPa), we find $2r_p = 189$ μm from (1.4.1). The resulting r_p is comparable to the characteristic dimensions of the Hertz contact zones. Satisfaction of estimate (1.4.2) will therefore lead to the breaking of one of the conditions validating the asymptotic analysis [1, 2], i.e., the condition that $\delta_0 = \ell/\tilde{b} \ll 1$. However, when we take into consideration that the rate of crack growth at the break is very high, and that the time spent on the break is small, it is possible, with adequate accuracy for practical purposes, to leave out of the calculation the violation of the condition $\delta_0 \ll 1$. In other words, we will estimate (1.4.2) to be satisfied throughout the entire process of fatigue crack development, carrying out the calculations in accordance with the asymptotic formulas from [1, 2] in the assumption that $\delta_0 \ll 1$ applies also throughout the entire destruction process.

Thus, as we study the processes of contact fatigue we can make use, with accuracy adequate for practical purposes, of the linear mechanics of quasibrittle destruction.

2. Mathematical Model of the Contact Fatigue Process. The statistical model of destruction must include both a description of the elementary acts of destruction, i.e., the process of fatigue crack development, as well as the statistics for the given elementary destruction occurrences. Examples of such an approach, oriented toward use in structures, can be found in [15, 16]. Moreover, in fatigue analysis we frequently resort to the concept of material damage [16].

2.1. The elementary destruction event. A rather complete review of the kinetic equations for the development of fatigue cracks can be found in [4]. In general form, the study of the development of a fatigue crack reduces to the solution of the Cauchy problem

$$dl/dN = g(k_1),\ l|_{N=0} = l_0 \tag{2.1.1}$$

(the function g for each specific form of the kinetic equation has its own form and N represents the number of loading cycles).

In the case of contact fatigue k_1 is determined from solution of the contact problem for elastic bodies subjected to cracking. When we take into consideration the assumption as to the smallness of the dimensions of the fatigue crack in comparison to the dimensions of the contact region, we find [1, 2] that the coefficients of intensity at the tips of the crack are quite close to one another and, on the whole, can be presented in the form

$$k_1 = k_{10} l^{1/2} \tag{2.1.2}$$

[k_{10} depends on N and α, the coordinates for the location of the crack center (x, y), and is independent of ℓ].

According to [12, 17], the direction of fatigue-crack development (the angle α) after a relatively few cycles can be found from the equation

$$k_2(N,\ x,\ y,\ \alpha) = 0, \tag{2.1.3}$$

where the coefficient of shearing stress intensity k_2 will also be approximated by the single-term asymptotic expansion [1, 2], calculated at the point (x, y). Having solved (2.1.3) for α, we have two angles α_1 and α_2: $\alpha_2 - \alpha_1 = \pi/2$. In this case, for the solution we will select the angle at which k_1 is at its maximum. We will assume the crack to be rectilinear throughout the entire process of its development. This assumption is in agreement with the hypothesis of crack smallness in comparison to the dimension of the contact region and, consequently, owing to the expression for k_2 [1, 2] in the single-term approximation with a constant load amplitude, where the angle α is independent of N, i.e., $\alpha = \alpha(x, y)$. We will subsequently assume that the independence of α from N exists also in the case of a time-variable load amplitude. In this case, for the $\alpha = \alpha(x, y)$ we can choose, for example, a value averaged in some fashion and derived from the angles $\alpha_i = \alpha_i(x, y)$, corresponding to loads with i-th amplitudes.

If we take into consideration that (2.1.1) includes the maximum k_1 for the cycle when $\alpha = \alpha(x, y)$, we will assume (2.1.1) to have been written for $\max_x k_1$.

2.2. The probability of destruction. Let us assume that the probability of this process proceeding for some number of cycles without destruction by some component part (the specimen) is a function of the local destruction within the material.

Let the probability of an absence of destruction at the point (x, y) on elapse of N cycles be p(N, x, y). Then

$$p(N+\Delta N,\ x,\ y) = p_0(N,\ \Delta N,\ x,\ y)p(N,\ x,\ y), \tag{2.2.1}$$

where $p_0(N, \Delta N, x, y)$ is the conditional probability for the absence of destruction at the point (x, y) within ΔN cycles, beginning from a state corresponding to the elapse of N loading cycles. We will relate the probability $p_0(N, \Delta N, x, y)$ with the accumulation of damage in the following manner:

$$p_0(N,\ \Delta N,\ x,\ y) = 1 - \Delta N v(N,\ x,\ y) \tag{2.2.2}$$

[v(N, x, y) is the rate of material damage accumulation at the point (x, y) on elapse of N cycles]. When we substitute (2.2.2) into (2.2.1) and approach the limit $\Delta N \to 0$, we obtain

$$p(N, x, y) = p(0, x, y) \exp\left[-\int_0^N v(s, x, y)\, ds\right], \tag{2.2.3}$$

p(0, x, y) is the probability of the absence of destruction at the point (x, y) at the beginning of the loading process, for which, without loss of generality, we assume

$$p(0, x, y) = \exp\left[-\int_{-\infty}^0 v(s, x, y)\, ds\right]. \tag{2.2.4}$$

Thus, the problem reduces to the determination solely of the damage to the material $\int_{-\infty}^{N} v\, ds$ for any given N at each point (x, y).

2.3. Material damage and its probability p(N, x, y). Let us introduce three scales of length: L_0, L_1, and L_2. We will assume L_0 to be commensurate with the characteristic dimension of the problem, e.g., with the dimension of the contact region, that L_2 is commensurate with the characteristic dimension of the microнонuniformities of the material (for steel this may be the dimension of the martensitic spicules), while we choose L_1 in a manner such that $L_2 \ll L_1 \ll L_0$ and that within the volume of the $L_1{}^3$ material there be present a sufficiently large number of defects (cracks). We will subsequently make no distinction between the cracks themselves and the various kinds of defects within the material that lead to fatigue cracks.

Let us examine a volume of material with the characteristic linear dimension L_1, identifying it with the material point. We will introduce the concept of the density probability function for the distribution of the number of cracks on the basis of the ℓ dimensions in a single volume with its center at the point (x, y) after N loading cycles have elapsed, and we will denote this function $f(N, x, y, \ell)$. The number of cracks in a unit volume with length in the range $[\ell, \ell + d\ell]$ is $f(N, x, y, \ell)d\ell$. Therefore,

$$\int_0^\infty f(N, x, y, l)\, dl = n(N, x, y) \tag{2.3.1}$$

[n(N, x, y) represents the crack density].

Let us assume that the cracks do not interact with one another, i.e., they are separated from each other through distances substantially in excess of their own dimensions. This results in a situation in which it is impossible for the cracks to merge into each other. We will also assume that the dimensions of the cracks, in the process of their development, remain substantially smaller than L_0 (see Sec. 1.4). Moreover, we will neglect the possibility of crack branching. By means of these assumptions we arrive at the conclusion that the quantity of cracks in the given volume is conserved in the process of their cyclical loading. Hence we find

$$f(N, x, y, l)dl = f(0, x, y, l_0)dl_0 \tag{2.3.2}$$

[ℓ is subject to the Cauchy problem (2.1.1)]. From (2.3.2) we obtain $f(N, x, y, \ell) = f(0, x, y, \ell_0)d\ell_0/d\ell$ and $n(N, x, y) = n(0, x, y)$.

The critical half length of the crack $\ell_k = \ell_k(N, x, y)$ will be that half length which at the point (x, y) gives us $k_1 = K_{fc}$. From (2.1.2) we then have

$$l_k = (K_{fc}/k_{10})^2. \tag{2.3.3}$$

By definition, when the crack reaches values of ℓ_k, catastrophic unstable destruction occurs. It is obvious that if at the point (x, y) there exist no cracks with a half length exceeding $\ell_k(N, x, y)$, then $\int_{l_k}^{\infty} f(N, x, y, l)dl = 0$ and $p(N, x, y) = 1$. Conversely, if at the points being analyzed the lengths of all cracks exceed $\ell_k(N, x, y)$, then $\int_{l_k}^{\infty} f(N, x, y, l)dl = n(0, x, y)$ and $p(N, x, y) = 0$, i.e., at the point (x, y) we have disintegration. In the general case, with an increase in the integral $\int_{l_k}^{\infty} f(N, x, y, l)dl$, $p(N, x, y)$ will diminish. The material damage $\int_{-\infty}^{N} v(s, x, y)ds$ is a monotonically increasing function of the integral $\int_{l_k}^{\infty} f(N, x, y, l)dl$. Consequently, from (2.2.3) and (2.2.4) we obtain

$$p(N, x, y) = \exp\left\{-G\left[\int_{l_k(N,x,y)}^{\infty} f(N, x, y, l)\, dl\right]\right\} \tag{2.3.4}$$

[G(x) is a monotonically increasing function of x; $G(0) = 0$, $G(n) = +\infty$]. Various approximations are possible for the function G, and of these the simplest is $G(x) = -\ln(1 - x/n)$. Here, with the aid of (2.3.1) and (2.3.4), we find

$$p(N, x, y) = \int_{0}^{l_k(N,x,y)} f(N, x, y, l)\, dl/n(0, x, y). \tag{2.3.5}$$

The determination in (2.3.5) of the probability $p(N, x, y)$ is most natural and is given by the initial distribution of the defects within the material. Indeed, let $\ell_{0k} = \ell_{0k}(N, x, y)$ be the initial half length of the crack at the point (x, y), and within N cycles of loading this will reach ℓ_k. Then, from (2.3.2) and (2.3.5) we will have

$$p(N, x, y) = \int_{0}^{l_{0k}(N,x,y)} f(0, x, y, l_0)\, dl_0/n(0, x, y). \tag{2.3.6}$$

Let us note that this model of fatigue destruction serves as the basis for the description of various forms of fatigue. Moreover, we should underscore that the two-dimensionality of the formulation in the construction of the statistical model of fatigue destruction has not, essentially, been employed anywhere. The analysis that we have carried out here, with trivial modifications, can be extended to the case of three measurements.

3. Wear and Disintegration. The processes of wear and disintegration are described in various ways, although fundamentally they are all based on the single mechanism of fatigue crack development. Fatigue disintegration is usually characterized by the probability P(N) of product utilization without failure over N loading cycles, while wear, as a rule, is evaluated on the basis of the depth of Y(N, x) at a given point on the surface at which the material has become worn (linear wear).

The products (foci) of the destruction will be characterized by two geometric parameters: by the thickness, which we will estimate from the depth y of the crack center and by the length that is related to the critical dimension of the crack:

$$\tilde{l}_k = \tilde{l}_k(N,\ x,\ y) = l_k(N,\ x,\ y - Y(N,\ x)). \tag{3.1}$$

For each point on the surface of the component part y = Y(N, x) we will introduce the set $\Omega_x = \Omega_x(N,\ y_*,\ \ell_*)$:

$$\Omega_x = \{y \mid 0 < Y(N, x) - y \leqslant y_*,\ \ \tilde{l}_k \leqslant l_*\}. \tag{3.2}$$

The number of disintegration products with a thickness of $0 < Y(N,\ x) - y \leq y_*$ and a length of $2\tilde{\ell}_k \leq 2\ell_*$ which becomes separated from a unit area Ω of the working surface will be defined as

$$I(N, y_*, l_*) = \int_\Omega dx \int_{\Omega_x(N, y_*, l_*)} n(0, x, y)\,[1 - \tilde{p}(N, x, y)]\, dy$$

[$\tilde{p}(N,\ x,\ y)$ is the probability of an absence of destruction at the point (x, y) after N loading cycles, when the surface of the component part is described by the equation y = Y(N, x)]. In this case, with the aid of (2.3.6) [or (2.3.5)], we have

$$\tilde{p}(N, x, y) = \int_0^{\tilde{l}_{0k}} f(0, x, y, l_0)\, dl_0 / n(0, x, y).$$

Here $\tilde{\ell}_{0k} = \tilde{\ell}_{0k}(N,\ x,\ y)$ is the half length of the crack at the initial instant of time, when after N loading cycles it reaches the value of $\tilde{\ell}_k$ from (3.1). Subsequently, we will understand wear to refer to the conditions under which

$$I(N, y_*, l_*) > I_w, \tag{3.3}$$

and we will understand disintegration to be represented by the conditions under which

$$I(N, +\infty, +\infty) - I(N, y_*, l_*) \leqslant I_p. \tag{3.4}$$

The constants ℓ_*, I_w, and I_p are determined experimentally.

In the process of wear the surface of a given component part will be subjected to displacement. It is obvious that this displacement of the boundaries of the body within ΔN loading cycles, resulting from wear and stratification, coincides with the increasing depth of the destroyed layer of the material within this time. Hence we find

$$\begin{gathered} Y(N, x) = m_c^{-1} \int_{\Omega_x} y\,[1 - \tilde{p}(N, x, y)]\, dy, \\ m_c = \int_{\Omega_x} [1 - \tilde{p}(N, x, y)]\, dy \end{gathered} \tag{3.5}$$

[y_*, included in (3.2), is found from (3.3)].

When we take into consideration the focus of the destruction and the independence of the realization of the disintegration effect at various points within the material of the component part, we arrive at the conclusion that the disintegration depends on the weakest element of the material. The probability of an absence of disintegration as represented by P(N) will therefore assume the form

$$P(N) = \min_{x;\, y \in C\Omega_x} \tilde{p}(N, x, y),$$

where the quantity y_* is determined from the criterion of fatigue wear (3.3), while $C\Omega_x$ represents an addition to Ω_x up to the interval $(-\infty,\ Y(N,\ x))$. Here, the disintegration process obviously occurs under conditions in which criterion (3.4) has been fulfilled.

4. Time-Variable and Stochastic Loading. Let us examine a periodic cyclical loading regime such that the amplitude of the maximum contact load assumes values of $\tilde{q}(N) = q_i$ for $kN_0 + n_i < N \le kN_0 + n_{i+1}$ $(i = 0, \ldots, j)$. Here N_0 represents the period of change in load; n_i are nonnegative numbers: $n_0 = 0$, $n_{j+1} = N_0$; $k = 0, 1, \ldots$

The method detailed above to describe contact fatigue can be extended to the case under consideration. In situations of practical importance N_0 is negligibly small in comparison to the number N of loading cycles within which linear wear or the probability of disintegration change by perceptible amounts. Therefore, with sufficient accuracy, we can determine the orientation angle α_i corresponding to the i-th amplitude of q_i from the following equation [see (2.1.3)]:

$$k_{2i}(\alpha_i) = 0, \; i = 0, \ldots, j, \tag{4.1}$$

while the resulting angle α_m of crack propagation can be determined from the following equation:*

$$\operatorname{tg}\alpha_m = \sum_{i=0}^{j} \Delta n_i k_{10i}^{2m} \sin\alpha_i \Big/ \sum_{i=0}^{j} \Delta n_i k_{10i}^{2m} \cos\alpha_i, \quad \Delta n_i = n_{i+1} - n_i. \tag{4.2}$$

The average value of Mk_{10} in the quantity k_{10}, which should be used in place of the latter throughout in the relationships of Secs. 1-3, can be taken in the form

$$Mk_{10} = \left\{\frac{1}{N_0}\left[\left(\sum_{i=0}^{j} \Delta n_i k_{10i}^{2m}\cos\alpha_i\right)^2 + \left(\sum_{i=0}^{j} \Delta n_i k_{10i}^{2m}\sin\alpha_i\right)^2\right]^{1/2}\right\}^{1/(2m)}. \tag{4.3}$$

In (4.2) and (4.3) the quantities k_{10i} are calculated by means of the angles α_i from (4.1). With a continuous change in $\tilde{q}(N)$ the sums in (4.2) and (4.3) are replaced by the appropriate integrals.

In the stochastic distribution of $\tilde{q}(t)$ with the probability density $f_q(t)$ relationships (4.1) and (4.3) have the following analogs:

$$\int_{\Omega_t} k_2(t, \alpha) f_q(t)\, dt = 0, \quad Mk_{10} = \left[\int_{\Omega_t} k_{10}^{2m}(t) f_q(t)\, dt\right]^{1/(2m)},$$

where Ω_t is the carrier $f_q(t)$; $k_{10}(t)$ and $k_2(t, \alpha)$ are the values of the coefficients k_{10} and k_2 when $\tilde{q} = \tilde{q}(t)$.

We should note that if the development of cracks is described by the Paris equation, then with a time-constant external load we can demonstrate that steady wear cannot be realized.

Thus, on the basis of our investigation into the single mechanism of fatigue-crack formation we initially formulated the statistical model of fatigue wear and disintegration, taking into consideration the original contamination of the material, its resistance to crack formation, and the contact and residual stresses.

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*Here 2m is the exponent in the Paris equation $d\ell/dN = g_0 k_1^{2m}$, to which the development of fatigue cracks is subject.

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AN ULTRASOUND METHOD FOR EXPERIMENTAL EVALUATION OF FIELD NONUNIFORMITIES IN INTERNAL DYNAMIC STRESSES

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The dynamically stressed state of machine elements and structures is determined through the measurement of the vibrations at the surfaces of these elements. Data relating to the structure of the elastic field within these elements are obtained through sequential calculations [1] based on mathematical relationships known to us from the theory of elasticity. These methods are based on measurements and calculations which have proved themselves in evaluating the structure of a static and quasistatic elastic field, but they become virtually useless when consideration must be given to the wavelike nature of the field. However, an increasing number of problems is encountered in engineering, where it is precisely these wave processes in machines and constructions that must be subjected to study [2]. There arises a need to find new principles for the experimental evaluation of field structure.

1. Let us turn to the studies [3, 4] where it is proposed to use the phenomenon of nonlinear interaction between elastic waves. The essence of this proposal lies in the fact that a plane monochromatic ultrasonic wave, on reaching a zone of a rather powerful

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